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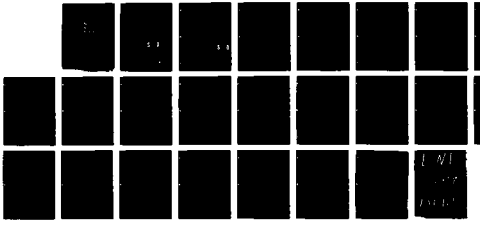

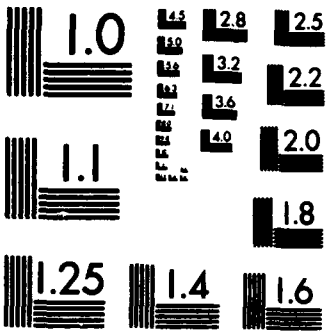

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APPROXIMATING MULTIVARIATE NORMAL ORTHANT PROBABILITIES USING THE CLARK ALGORITHM

Robert D. Gibbons
University of Illinois
R. Darrell Bock
University of Chicago
Donald Hedeker
University of Chicago

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Departments of Psychiatry and Biometry
University of Illinois at Chicago

and the

Illinois State Psychiatric Institute
1601 W. Taylor Street
Chicago, IL 60612

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$$\int_0^\infty \int_0^\infty \cdots \int_0^\infty \Phi_n(z_1, z_2, \ldots, z_n)\, \partial z_1, \partial z_2, \ldots, \partial z_n.$$

Exact results for this probability integral are unavailable for $m > 3$. Approximations for higher dimensional problems have generally yielded poor results except for *special cases*, such as compound symmetry, which is of limited value in practice. The purpose of this paper is to present a general approximation of this probability integral. The algorithm developed here is computationally tractable for $m = 50$ and accurate for very general correlational structures. The performance of this algorithm is compared to results based on Clark's (1961) original approximation, Gaussian quadrature formulae, and Monte Carlo simulation methods. Application of this approximation to problems of conditional dependence in IRT estimation problems is discussed.

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Approximating Multivariate Normal Orthant Probabilities using The Clark Algorithm

Robert D. Gibbons
University of Illinois

R. Darrell Bock
University of Chicago

Donald Hedeker
University of Chicago

Correspondence to be addressed to: Dr. R.D. Gibbons, Biometric Laboratory Illinois State Psychiatric Institute, 1601 W. Taylor St., Chicago, IL 60612, USA. Supported by grant # N00014-85-K-0586 from the Office of Naval Research.

ABSTRACT

The probability of m correlated random variables drawn from a multivariate normal distribution being non-negative is:

$$\int_0^\infty \int_0^\infty \cdots \int_0^\infty \Phi_n(x_1, x_2, \ldots, x_n)\, \partial x_1, \partial x_2, \ldots, \partial x_n.$$

Exact results for this probability integral are unavailable for $m > 3$. Approximations for higher dimensional problems have generally yielded poor results except for *special cases*, such as compound symmetry, which is of limited value in practice. The purpose of this paper is to present a general approximation of this probability integral. The algorithm developed here is computationally tractable for $m = 50$ and accurate for very general correlational structures. The performance of this algorithm is compared to results based on Clark's (1961) original approximation, Gaussian quadrature formulae, and Monte Carlo simulation methods. Application of this approximation to problems of conditional dependence in IRT estimation problems is discussed.

1 INTRODUCTION

With the renewed interest in marginal maximum likelihood methods (MML) in item response theory (Bock and Aitken, 1981), the necessity for approximating orthant probabilities of the multivariate normal distribution has arisen. In the MML estimation of item parameters, for example, all $s \leq 2^n$ orthant probabilities must be evaluated in order to obtain the likelihood of the parameters of a given IRT model. Similarly, the same multivariate normal orthant probabilities are required to obtain Bayesian estimates of ability assuming dichotomous scoring.

Because IRT models assume conditional independence among the items, that is, all association among the n items is completely explained by their joint association with the latent variables, m-dimensional Gaussian quadrature formulae have been used to provide the necessary orthant probability estimates. In this case, the simple "product formulae" for numerical integration can be applied, because the residuals of the model have the simple uncorrelated multivariate normal distribution $e \sim (0, \sigma_j^2 I)$, where σ_j^2 is the so-called item "uniqueness", that is, $1 - \alpha_j^2$, and α_j is the item factor loading. However, these numerical integrations become extremely expensive as the dimensionality of the problem increases. To the extent that the items have a simple factor structure, these approximations are accurate to any practical degree, however their behavior with more general correlational structures is predictably poor and the extent of bias introduced into parameter estimates is unknown.

In the general case, the probability that m correlated random variables drawn from a multivariate normal distribution are non-negative is:

$$\int_0^\infty \int_0^\infty \cdots \int_0^\infty \Phi_m(x_1, x_2, \ldots, x_m)\, \partial x_1, \partial x_2, \ldots, \partial x_m.$$

Exact results for the bivariate normal distribution have been obtained from the work of Sheppard (see Kendall and Stuart, 1973). David (1953) has shown that trivariate normal probabilities are a direct extension of Sheppard's earlier result. However, exact results for m-variate normal probabilities for $m > 3$ are unknown.

Kendall (1941), McFadden (1960) and Moran (1948) have developed infinite expansions of the quadrivariate normal integral, but these results are computationally cumbersome. More tractable representations have been proposed by Abrahamson (1964), Dutt (1973), Dutt and Lin (1975) and Childs (1967). Even in the quadrivariate case, however, these approximations are intractable for general correlational structures and have, therefore,

only been applied under restrictions of compound symmetry or band matrices in which $\rho = .5$ just off the main diagonal.

In a previous paper, we have developed an IRT-type model for estimating trend in correlated proportions (Gibbons and Bock, 1987). In this case the binary items are not components of the same test, but rather, dichotomous classifications ordered in time. Since the association between temporally proximal responses will typically be greater than the association between temporally distal responses, the assumption of conditional independence is untenable and residual autocorrelation must be assumed. At the suggestion of James Heckman (personal communication) we adapted the so-called Clark Algorithm (Clark, 1961) to the problem of evaluating the likelihood of this model. Our modified Clark Algorithm combines both Gaussian quadrature formulae with Clark's original approximation which is extremely fast. The purpose of this paper is to present details of this modified approximation, illustrate its computation, and determine its accuracy in a few relevant examples.

2 THE CLARK ALGORITHM

Designating positive directions 1 and negative directions 0, we may represent the probability of the positive orthant of an m-variate distribution by $P(1,1,\ldots,1)$, that of the negative orthant by $P(0,0,\ldots,0)$, and that of any one of the other $2^m - 2$ orthants by inserting the appropriate pattern of 1's and 0's. The Clark algorithm provides a computing approximate for any orthant of a multivariate normal distribution with arbitrary vector mean and covariance matrix. Clark (1961) derives the following formulas.

Let any three successive components from an m-variate vector, $\underset{\sim}{y}_1$, be distributed:

$$\begin{bmatrix} y_i \\ y_{i+1} \\ y_{i+2} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} \mu_i \\ \mu_{i+1} \\ \mu_{i+2} \end{bmatrix}, \begin{bmatrix} \sigma_i^2 & & \\ \sigma_i\sigma_{i+1}\rho_{i,i+1} & \sigma_{i+1}^2 & \\ \sigma_i\sigma_{i+2}\rho_{i,i+2} & \sigma_{i+1}\sigma_{i+2}\rho_{i+1,i+2} & \sigma_{i+2}^2 \end{bmatrix} \right)$$

Let $\tilde{y}_i = \max(y_i) = y_i$, and compute the probability that $y_{i+1} > \tilde{y}_i$ as follows:

$$\text{set} \qquad z_{i+1} = (\mu_i - \mu_{i+1})/\varsigma_{i+1},$$

$$\text{where} \qquad \varsigma_{i+1}^2 = \sigma_i^2 + \sigma_{i+1}^2 - 2\sigma_i\sigma_{i+1}\rho_{i,i+1}.$$

$$\begin{aligned} \text{Then} \quad P(y_{i+1} > \tilde{y}) &= P(y_{i+1} - \tilde{y} > 0) \\ &= \Phi(-z_{i+1}) \end{aligned}$$

the value of the univariate normal distribution function at the standard deviate $-z_{i+1}$.

Now let $\tilde{y}_{i+1} = \max(y_i, y_{i+1})$ and assume (as an approximation) that $(y_{i+2}, \tilde{y}_{i+1})$ is bivariate normal with means,

$$\begin{aligned} \mu(y_{i+2}) &= \mathcal{E}(y_{i+2}) = \mu_{i+2} \\ \mu(\tilde{y}_{i+1}) &= \mathcal{E}(\tilde{y}_{i+1}) = \mu_i\Phi(z_{i+1}) + \mu_{i+1}\Phi(-z_{i+1}) + \varsigma_{i+1}\phi(z_{i+1}), \end{aligned}$$

variances

$$\begin{aligned} \sigma^2(y_{i+2}) &= \mathcal{E}(y_{i+2}^2) - \mathcal{E}^2(y_{i+2}) = \sigma_{i+2}^2, \\ \sigma^2(\tilde{y}_{i+1}) &= \mathcal{E}(\tilde{y}_{i+1}^2) - \mathcal{E}^2(\tilde{y}_{i+1}), \end{aligned}$$

where

$$\mathcal{E}(\tilde{y}_{i+1}^2) = (\mu_i^2 + \sigma_i^2)\Phi(z_{i+1}) + (\mu_{i+1}^2 + \sigma_{i+1}^2)\Phi(-z_{i+1}) + (\mu_i + \mu_{i+1})\varsigma_{i+1}\phi(z_{i+1}),$$

and correlation

$$\rho(\tilde{y}_{i+1}, y_{i+2}) = \frac{\sigma_i\rho_{i,i+2}\Phi(z_{i+1}) + \sigma_{i+1}\rho_{i+1,i+2}\Phi(-z_{i+1})}{\sigma(\tilde{y}_{i+1})}.$$

Then,

$$P(y_{i+2} = \max(y_i, y_{i+1}, y_{i+2})) = P((y_{i+2} - y_{i+1} > 0) \cap (y_{i+2} - y_i > 0))$$

is approximated by

$$\begin{aligned} P(y_{i+2} > \tilde{y}_{i+1}) &= P(y_{i+2} - \tilde{y}_{i+1} > 0) \\ &= \Phi\left(\frac{\mu_{i+2} - \mu(\tilde{y}_{i+1})}{\sqrt{\sigma_{i+2}^2 + \sigma^2(\tilde{y}_{i+1}) - 2\sigma_{i+2}\sigma(\tilde{y}_{i+1})\rho(\tilde{y}_{i+1}, y_{i+2})}}\right) \end{aligned}$$

Assuming as a working approximation that $\tilde{y}_{i+1}$ is normally distributed with the above mean and variance, we may therefore proceed, recursively from $i = 1$ to $i = m - 1$, where y_{m+1} is an independent dummy variate with mean zero and variance zero (i.e. $y_{m+1} = 0$).

Then

$$\begin{aligned} & P(y_{m+1} = \max(y_1, y_2, \ldots, y_{m+1})) \\ &= P((y_{m+1} - y_1 > 0) \cap (y_{m+1} - y_2 > 0) \cap \ldots \cap (y_{m+1} - y_m > 0)) \\ &= P((-y_1 > 0) \cap (-y_2 > 0) \cap \ldots \cap (-y_m > 0)) \end{aligned}$$

approximates the probability of the negative orthant of the specified multivariate normal distribution. The probability of any other orthant can be obtained by reversing the signs of the variates corresponding to 1's in the orthant pattern.

3 COMPUTATIONAL EXAMPLE

For computational purposes, it is convenient to set $\Phi(-z) = 1 - \Phi(z)$ and rewrite Clark's equations as:

$$\mu(\tilde{y}_{i+1}) = \mu_{i+1} + (\mu_i - \mu_{i+1})\Phi(z_{i+1}) + \varsigma_{i+1}\phi(z_{i+1}),$$

$$\begin{aligned} \mathcal{E}(\tilde{y}_{i+1}^2) &= \mu_{i+1}^2 + \sigma_{i+1}^2 + (\mu_i^2 + \sigma_i^2 - \mu_{i+1}^2 - \sigma_{i+1}^2)\Phi(z_{i+1}) \\ &\quad + (\mu_i + \mu_{i+1})\varsigma_{i+1}\phi(z_{i+1}), \end{aligned}$$

$$\sigma^2(\tilde{y}_{i+1}) = \mathcal{E}(\tilde{y}_{i+1}^2) - \mathcal{E}^2(\tilde{y}_{i+1}),$$

$$\begin{aligned} \sigma^2(\tilde{y}_{i+1}, y_{i+2}) &= \sigma^2(y_{i+1}, y_{i+2}) + [\sigma^2(\tilde{y}_i, y_{i+2}) \\ &\quad - \sigma^2(y_{i+1}, y_{i+2})]\Phi(z_{i+1}) \end{aligned}$$

Given this transformation, the ith step now only requires a single evaluation of $\Phi(.)$ and $\phi(.)$. For example, suppose that

$$\underline{\mu} = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} 3 & & \\ 1 & 2 & \\ 1 & 0 & 2 \end{bmatrix}$$

Given these values for $\underline{\mu}$ and Σ, the probability of the negative orthant (- - -) can be obtained as follows.

First find $P(y_3 > \tilde{y}_2)$ as:

$$\begin{aligned} \varsigma_2 &= \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}^2} \\ &= \sqrt{3 + 2 - 2(1)} = 1.732 \end{aligned}$$

$$\begin{aligned} z_2 &= (\mu_1 - \mu_2)/\varsigma_2 \\ &= (3-2)/1.732 = .577 \end{aligned}$$

$$\begin{aligned} \mathcal{E}(\tilde{y}_2) &= \mu_2 + (\mu_1 - \mu_2)\Phi(z_2) + \varsigma_2\phi(z_2) \\ &= 2 + (3-2)\Phi(.577) + 1.732\phi(.577) = 3.30 \end{aligned}$$

$$\begin{aligned} \mathcal{E}(\tilde{y}_2^2) &= \mu_2^2 + \sigma_2^2 + (\mu_1^2 + \sigma_1^2 - \mu_2^2 - \sigma_2^2)\Phi(z_2) \\ &\quad +(\mu_1 + \mu_2)\varsigma_2\phi(z_2) \\ &= 6 + 6\Phi(.577) + 5(1.732)\phi(.577) = 13.23 \end{aligned}$$

$$\begin{aligned} \sigma^2(\tilde{y}_2) &= \mathcal{E}(\tilde{y}_2^2) - \mathcal{E}^2(\tilde{y}_2) \\ &= 13.23 - 3.30^2 = 2.34 \end{aligned}$$

$$\begin{aligned} \sigma^2(\tilde{y}_2, y_3) &= \sigma_{23}^2 + (\sigma_{13}^2 - \sigma_{23}^2)\Phi(z_2) \\ &= \Phi(.577) = .72 \end{aligned}$$

Therefore;

$$\begin{aligned} P(y_3 > \tilde{y}_2) &= \Phi\left(\frac{\mu_3 - \tilde{\mu}_2}{\sqrt{\sigma_3^2 + \tilde{\sigma}_2^2 - 2\tilde{\sigma}_{23}^2}}\right) \\ &= \Phi\left(\frac{2 - 3.3}{\sqrt{2 + 2.34 - 2(.72)}}\right) = .222 \end{aligned}$$

To determine $P(\tilde{y}_3 > 0)$ we set $y_4 = 0$ and $\sigma_4^2 = 0$, and compute:

$$
\begin{aligned}
\varsigma_3 &= \sqrt{\tilde{\sigma}_2^2 + \sigma_3^2 - 2\tilde{\sigma}_{23}^2} \\
&= \sqrt{2.34 + 2 - 2(.72)} = 1.703
\end{aligned}
$$

$$
\begin{aligned}
z_3 &= (\tilde{\mu}_2 - \mu_3)/\varsigma_3 \\
&= (3.3 - 2)/1.703 = .763
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{E}(\tilde{y}_3) &= \mu_3 + (\tilde{\mu}_2 - \mu_3)\Phi(z_3) + \varsigma_3\phi(z_3) \\
&= 2 + (3.3 - 2)\Phi(.763) + 1.703\phi(.763) = 3.518
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{E}(\tilde{y}_3^2) &= \mu_3^2 + \sigma_3^2 + (\tilde{\mu}_2^2 + \tilde{\sigma}_2^2 - \mu_3^2 - \sigma_3^2)\Phi(z_3) \\
&\quad +(\tilde{\mu}_2 + \mu_3)\varsigma_3\phi(z_3) \\
&= 6 + 7.23\Phi(.763) + 5.3(1.703)\phi(.763) = 14.307
\end{aligned}
$$

$$
\begin{aligned}
\sigma^2(\tilde{y}_3) &= \mathcal{E}(\tilde{y}_3^2) - \mathcal{E}^2(\tilde{y}_3) \\
&= 14.307 - 3.518^2 = 1.931
\end{aligned}
$$

$$
\begin{aligned}
\sigma^2(\tilde{y}_3, y_4) &= \sigma_{34}^2 + (\sigma_{14}^2 - \sigma_{34}^2)\Phi(z_3) \\
&= 0
\end{aligned}
$$

Therefore;

$$
\begin{aligned}
P(y_4 > \tilde{y}_3) &= \Phi\left(\frac{\mu_4 - \tilde{\mu}_3}{\sqrt{\sigma_4^2 + \tilde{\sigma}_3^2 - 2\tilde{\sigma}_{34}^2}}\right) \\
&= \Phi\left(\frac{0 - 3.518}{\sqrt{0 + 1.931 - 2(0)}}\right) = \Phi(-2.53)
\end{aligned}
$$

Hence, $P(0 > \tilde{y}_3) = \Phi(-2.53) = .006$, which is the probability of orthant (- - -).

4 THE MODIFIED CLARK ALGORITHM #1

In our previous paper (Gibbons and Bock, 1987), we noted that the accuracy of the Clark approximation diminishes with increasing magnitude of

the correlations. If we apply the Clark approximation directly to estimates of inter-item correlations, it will generally yield biased results due to the size of correlations. This is true regardless of whether the correlation matrix exhibits the property of conditional independence. Alternatively, if we examine the residual inter-item correlation matrix at fixed points on the ability scale, we will observe the identity matrix for conditionally independent solutions or small residual correlations for those item pairs that are conditionally dependent. In light of this, we evaluate the response function at several fixed points on the ability scale using Gauss-Hermite quadrature, and correct these estimates using the Clark algorithm. These corrections depend only on the residual inter-item correlations, which in practice should be quite small. The first modified Clark algorithm proceeds as follows.

Step 1 Obtain a factor solution of dimension m, using full information factor analysis for binary data (Bock and Aitken, 1981, and Bock, Gibbons and Muraki, 1986) if the correlations are unknown or using principal factor analysis if the item-correlations are known, as in the following simulations.

Step 2 Using the estimated factor loading matrix $\underset{n \times k}{\lambda}$ compute the estimated residual correlation matrix $R'_{n \times n}$ as $\rho'_{ij} = \sum_{l=t}^{m} \lambda_{li}\lambda_{lj}$ for $i \neq j$ else $\rho'_{ij} = 1$, where t is the dimensionality of the space we are conditioning on plus 1. For example, in the unidimensional case, $t = 2$. This correlation matrix represents the degree of residual conditional dependence.

Step 3 Given the previous values of item thresholds γ_j and item factor loadings λ_{ij} for the $t-1$ prinical factors, compute the invariant item parameters a_j (slope) and b_j (intercept). For example, in the unidimensional case,

$$a_j = \lambda_j / \sqrt{1 - \lambda_j^2}$$

and

$$b_j = -\gamma_j / \sqrt{1 - \lambda_j^2}$$

Step 5 At each point on the ability dimension (i.e. at each quadrature node X_k) compute the value of the response function for each item as:

$$z_{jk} = c_j + a_j X_k$$

where $c_j = -a_j b_j$ and X_k are the nodes of the Gauss-Hermite polynomial (see Stroud and Sechrest, 1966).

Step 6 At each quadrature point, substitute the values of z_{jk} for the mean vector $\underline{\mu}$ and R' for the covariance matrix Σ and compute the Clark approximated probability $C_l(X_k)$. Accumulating these probabilities over all quadrature nodes for a given response pattern ($\underline{x}_l$) yields the desired marginal probability estimate

$$\begin{aligned} h(\underline{x}_l) &= \int_{-\infty}^{\infty} C_l(\theta)\phi(\theta)\partial(\theta) \\ &= \sum_{k=1}^{q} C_l(X_k)A(X_k) \end{aligned}$$

where $A(X_k)$ is the corresponding weight at quadrature node X_k.

We note that in practice, the effect of assuming normality of the maximum of two jointly normal variables, produces probability overestimates in the tails of the distribution. As such, we apply an empirically based correction factor to these probability estimates which involves raising the Clark adjusted probability estimate to the power 1.3. This correction factor appears to provide the neccessary adjustment across the entire quadrature space.

5 THE MODIFIED CLARK ALGORITHM #2

Algorithm #1 is computationally expensive, because the Clark approximation must be evaluated for each response pattern at every point in the quadrature space. In a problem with 1000 unique response patterns and two principal factors, each with 10 quadrature points, the Clark approximation must be invoked 100,000 times per iteration. An alternate approach is to simply use the Clark approximated probability as a correction term, applied directly to the usual probability estimate obtained from the quadrature solution; that is:

$$\begin{aligned} h(\underline{x}_l) &= C_l \int_{-\infty}^{\infty} P_l(\theta)\phi(\theta)\partial(\theta) \\ &= C_l \sum_{k=1}^{q} P_l(X_k)A(X_k) \end{aligned}$$

where

$$
\begin{aligned}
P_l &= P(x_1, \ldots, x_n) \\
&= \int_{-\infty}^{\infty} [\prod_{j=1}^{n} [p_j(\theta)]^{z_j} [1 - p_j(\theta)]^{1-z_j}] \phi(\theta) d(\theta) \\
&= \sum_{k=1}^{q} [\prod_{j=1}^{n} p_j(X_k)^{z_j} [1 - p_j(X_k)]^{1-z_j}] A(X_k)
\end{aligned}
$$

Returning to the previous example of 1000 unique response patterns and 10 quadrature points in each of two dimensions (i.e. 100 points in total), the Clark algorithm need only be invoked 1000 times per iteration; that is, once for each response pattern in contrast to 100,000 times for algorithm #1.

The probabilities obtained using algorithm #2 differ from those obtained from algorithm #1 in that they will not sum to unity even if all response patterns are realized in the sample. When the number of items is small, say ten or less, these probabilities can be normalized to yield the appropriate metric; however, in larger problems (i.e. with eleven or more items) normalization is not possible, because all patterns generally are not realized in even large samples. We note, however, that maximum likelihood estimation does not require normalized probabilities as long as their relative magnitudes are invariant to transformation of scale. This condition does hold for algorithm #2.

6 ILLUSTRATION

To examine the accuracy of the modified Clark Algorithm, we designed the following limited simulation study. First, we simulated one million five-variate normal deviates for each of the following conditions:

1) compound symmetric matrices with $\rho = 0.2$ through $\rho = 0.8$. For example.

$$
R = \begin{bmatrix}
1.0 & & & & \\
0.5 & 1.0 & & & \\
0.5 & 0.5 & 1.0 & & \\
0.5 & 0.5 & 0.5 & 1.0 & \\
0.5 & 0.5 & 0.5 & 0.5 & 1.0
\end{bmatrix}
$$

2) Autocorrelated matrices with $\rho = 0.2$ through $\rho = 0.8$. For example,

$$R = \begin{bmatrix} 1.0 & & & & \\ 0.5 & 1.0 & & & \\ 0.25 & 0.5 & 1.0 & & \\ 0.125 & 0.25 & 0.5 & 1.0 & \\ 0.0625 & 0.125 & 0.25 & 0.5 & 1.0 \end{bmatrix}$$

9) Conditionally dependent correlation matrices with principal factor loadings of $\lambda_{11} = \lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{15}$ ranging from 0.5 to 0.7 and two method related factors, the first with $\lambda_{21} = \lambda_{22} = \lambda_{23}$ ranging from 0.2 through 0.7 and $\lambda_{24} = \lambda_{25} = 0.0$, and the second with $\lambda_{31} = \lambda_{32} = \lambda_{33} = 0.0$ and $\lambda_{34} = \lambda_{35}$ ranging from 0.2 through 0.7.

For example, the correlation matrix corresponding to factor pattern matrix:

$$\lambda = \begin{bmatrix} 0.7 & 0.3 & 0.0 \\ 0.7 & 0.3 & 0.0 \\ 0.7 & 0.3 & 0.0 \\ 0.7 & 0.0 & 0.3 \\ 0.7 & 0.0 & 0.3 \end{bmatrix}$$

is

$$R = \begin{bmatrix} 1.0 & & & & \\ 0.58 & 1.0 & & & \\ 0.58 & 0.58 & 1.0 & & \\ 0.49 & 0.49 & 0.49 & 1.0 & \\ 0.49 & 0.49 & 0.49 & 0.58 & 1.0 \end{bmatrix}.$$

In each of the above simulated conditions, the mean vectors were zero, therefore, the binary response patterns were obtained by dichotomizing the simulated normal deviates at zero. The dichotomized response patterns were then sorted and unique patterns and their respective frequencies were accumulated. When divided by one million, these frequencies yield the so called Monte Carlo probability estimates which should be exact to at least four decimal places; that is, the standard error of the simulated probabilities is:

$$\left(\frac{p_j(1-p_j)}{N}\right)^{\frac{1}{2}}$$

which has a maximum value of:

$$\left(\frac{.5(1-.5)}{1000000}\right)^{\frac{1}{2}} = .0005$$

Accuracy of the estimated probabilities was determined by computing the average absolute deviation for each method (i.e. the average absolute difference between Monte Carlo and Clark estimates over all response patterns).

In an effort to examine the properties of these approximations in larger sets of items, that are more typical in practice, we produced two 10-item simulations. The first, consisted of ten million multivariate normal deviates for the compound symmetric case ($\rho = 0.5$), and the second for the conditionally dependent example with principal factor loadings of $\lambda_{1j} = 0.6$ and method related factor loadings of $\lambda_{2,1} \ldots \lambda_{2,10} = 0.4$ and $\lambda_{3,11} \ldots \lambda_{3,20} = 0.4$.

7 RESULTS

Results of the simulations are displayed in Tables 1-5. Inspection of Table 1, which displays results for the five-item compound symmetric case, reveals that on average, both the quadrature solution and the modified Clark algorithm #2, recover the Monte Carlo estimates to four decimal places (i.e. average difference $\rho = 0.5$, .0002) whereas the modified Clark algorithm #1 is slightly less accurate (i.e. average difference $\rho = 0.5$, .0009). In contrast, the original Clark algorithm is inferior to the other methods and this inferiority increases with larger correlations ($\rho = 0.8$, average difference = .0163). These results are exactly as expected for a conditionally independent correlational structure. In contrast, inspection of Table 2 reveals that autocorrelation produces inferior estimates for the quadrature solution (eg. $\rho = 0.5$, average difference = .0070) whereas the modified Clark algorithms produced more consistent estimates across the entire range (eg. $\rho = 0.5$, average difference for algorithm #1 = .0041 and for algorithm #2 = .0036). Algorithm #2 was in general, slightly better than algorithm #1. The original Clark algorithm produced reasonable estimates through $\rho = 0.5$, but deteriorated quickly for values of $\rho > 0.5$. Since the elements of the correlation matrix become more homogeneous for extreme values of ρ (eg. $\rho = 0.8$) it is not surprising that the performance of the quadrature solution stabilized for values of $\rho > 0.6$.

In terms of five-item conditionally dependent correlational structures (see Table 3), the modified Clark algorithms performed similarly whereas the original Clark algorithm and the quadrature solution were consistently

inferior, for even moderate dependence (λ_{2j} or $\lambda_{3j} > 0.3$). The modified Clark algorithm #2 produced average differences that were slightly smaller than algorithm #1 and as little as one-sixth the size of the standard quadrature solution. Overall, performance was better for solutions in which the principal factor loadings were smaller (eg. $\lambda_{11} = 0.5, \lambda_{21} = 0.5, \lambda_{31} = 0.0$, average difference for algorithm #1 = .0034, average difference for algorithm #2 = .0031, average difference for original Clark = .0041 and average difference for quadrature = .0084 versus $\lambda_{11} = 0.7, \lambda_{21} = 0.5, \lambda_{31} = 0.0$, algorithm #1 = .0068, algorithm #2 = .0048, original Clark = .0091, and quadrature =.0099).

Results for the larger item-sets are presented in Table 4 for the compound symmetric case and Table 5 for the conditional dependent case. In general, results for the larger item-sets parallel those of the smaller item-sets. For the compound symmetric case ($\rho = 0.5$) the average differences were .000065 for algorithm #1, .000007 for algorithm #2, .000007 for the quadrature solution. For the conditionally dependent case ($\lambda_{1j} = 0.6, \lambda_{2j} = 0.4$ or 0.0 and $\lambda_{3j} = 0.4$ or 0.0), the average differences were .000218 for algorithm #1, .000242 for algorithm #2, .000329 for the quadrature solution.

8 DISCUSSION

The results of this study clearly demonstrate that when the assumption of conditional independence is violated, bias in the standard IRT probability estimates are produced. Both modified Clark algorithms presented here minimize this bias; however, algorithm #2 produces slightly more accurate results than algorithm #1 for small numbers of items, at a remarkable savings in computation. Algorithm #2 decreases the required computation by a factor of q^m, where m is the number of underlying dimensions and q is the number of quadrature points in each dimension. When compared to the standard quadrature solution, algorithm #2 requires an additional s evaluations of the Clark algorithm, where s is the number of uniquely observed response patterns. Conversely, when method related factors do exist, multiple-factor solutions will greatly increase the computational complexity of the standard IRT approach, whereas no increase in computation is required for algorithm #2.

It is important to point out that the use of these approximations should not replace multiple factor solutions where the additional factors contribute to our understanding of individual differences. Indeed, the methods described here treat these potentially meaningful effects as errors of measure-

ment. By allowing for more general types of measurement variability, these methods can mask important characteristics of item-person interactions. There are, however, situations where small method related effects confound our ability to accurately characterize the dominant dimension of interest, because the inter-item association is not strictly a function of a single underlying trait. In these cases, the ability to segment these more complex measurement errors from our estimates of the central trait or aptitude of interest is a highly desirable goal of measurement.

Our future research in the application of the Clark algorithm for likelihood evaluation of IRT models, will focus on the estimation of ability. When method related factors violate the assumption of conditional independence, current methods for estimating ability (Bock and Aitken, 1981) will produce incorrect results. Some preliminary work in this area suggests that the variance in Bayes "expected *a posteriori*" estimates (EAP) for fixed levels of ability, increases by a factor of 2 to 3 in the presence of even moderate dependence. Substituting Clark estimates for the conditional probabilities that are usually employed in calculating EAP estimates should account for this "extra" normal variability, and therefore, provide more accurate ability estimates.

In addition, we will also further explore the difference between our two modified Clark algorithms, by focusing on their behavior at individual points in the quadrature space. In this way, we hope to even further improve the performance of algorithm #1 relative to algorithm #2, and perhaps, develop a third procedure which is an improvement over both.

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TABLE 1
COMPOUND SYMMETRY

	Average Difference			
r	Algorithm #1	Algorithm #2	Quadrature	Clark
.2	0.0007	0.0003	0.0003	0.0022
.3	0.0011	0.0003	0.0003	0.0034
.4	0.0019	0.0002	0.0003	0.0049
.5	0.0009	0.0002	0.0003	0.0068
.6	0.0016	0.0002	0.0002	0.0091
.7	0.0022	0.0002	0.0002	0.0122
.8	0.0006	0.0006	0.0007	0.0163

TABLE 2
AUTOCORRELATION

	Average Difference			
ρ	Algorithm #1	Algorithm #2	Quadrature	Clark
.2	0.0013	0.0013	0.0037	0.0017
.3	0.0022	0.0021	0.0051	0.0026
.4	0.0031	0.0028	0.0062	0.0037
.5	0.0040	0.0036	0.0070	0.0050
.6	0.0050	0.0044	0.0075	0.0067
.7	0.0054	0.0051	0.0075	0.0090
.8	0.0060	0.0054	0.0071	0.0121

Table 3
CONDITIONAL DEPENDENCE

		Average Difference			
λ_1	CD	Algorithm #1	Algorithm #2	Quadrature	Clark
.5	.2	0.0011	0.0005	0.0014	0.0023
	.3	0.0016	0.0011	0.0031	0.0024
	.4	0.0022	0.0020	0.0054	0.0031
	.5	0.0034	0.0031	0.0084	0.0041
	.6	0.0047	0.0045	0.0121	0.0057
	.7	0.0075	0.0065	0.0173	0.0083
.6	.2	0.0018	0.0006	0.0015	0.0034
	.3	0.0025	0.0014	0.0032	0.0037
	.4	0.0031	0.0025	0.0058	0.0046
	.5	0.0047	0.0039	0.0089	0.0060
	.6	0.0070	0.0058	0.0130	0.0081
	.7	0.0123	0.0083	0.0204	0.0120
.7	.2	0.0015	0.0008	0.0016	0.0052
	.3	0.0026	0.0018	0.0036	0.0057
	.4	0.0041	0.0031	0.0063	0.0070
	.5	0.0068	0.0048	0.0099	0.0091
	.6	0.0117	0.0073	0.0159	0.0125
	.7	0.0242	0.0170	0.0305	0.0229

Table 4
COMPOUND SYMMETRY
$\rho = 0.5$
25 PATTERNS FOR 10 ITEMS

Pattern	Algorithm #1	Algorithm #2	Quadrature	Monte Carlo
1111111111	.097932	.091086	.090469	.090958
1111111110	.009606	.009115	.009095	.009055
1111111101	.009229	.009119	.009095	.009049
1111111100	.002018	.002015	.002018	.002011
1111111011	.008887	.009122	.009095	.009094
1111111010	.001965	.002016	.002018	.002024
1111111001	.001980	.002017	.002018	.002008
1111111000	.000793	.000758	.000761	.000758
1111110111	.008578	.009125	.009095	.009077
1111110110	.001917	.002017	.002018	.002021
1111110101	.001930	.002018	.002018	.002013
1111110100	.000780	.000758	.000761	.000759
1111110011	.001948	.002019	.002018	.002018
1111110010	.000779	.000758	.000761	.000747
1111110001	.000781	.000758	.000761	.000769
1111110000	.000478	.000432	.000434	.000436
1111101111	.008302	.009126	.009095	.009126
1111101110	.001874	.002017	.002018	.002050
1111101101	.001885	.002018	.002018	.002015
1111101100	.000768	.000758	.000761	.000741
1111101011	.001901	.002019	.002018	.002040
1111101010	.000767	.000758	.000761	.000753
1111101001	.000768	.000759	.000761	.000750
1111101000	.000473	.000432	.000434	.000433
1111100111	.001924	.002019	.002018	.002037
REMAINING PATTERNS	.831716	.836967	.837527	.837249
AVERAGE DIFFERENCE	.000065	.000007	.000007	

Table 5
CONDITIONAL DEPENDENCE
$\lambda_{1j} = 0.6$
λ_{2j} and $\lambda_{3j} = 0.4$ or 0.0
25 PATTERNS FOR 10 ITEMS

Pattern	Algorithm #1	Algorithm #2	Quadrature	Monte Carlo
1111111111	.058630	.099093	.049837	.071364
1111111110	.008145	.010562	.007744	.008396
1111111101	.007586	.010753	.007744	.008384
1111111100	.002818	.002365	.002115	.002726
1111111011	.007119	.010809	.007744	.008387
1111111010	.002676	.002399	.002115	.002707
1111111001	.002645	.002432	.002115	.002743
1111111000	.001620	.000993	.000889	.001767
1111110111	.006694	.010750	.007744	.008432
1111110110	.002594	.002414	.002115	.002732
1111110101	.002525	.002429	.002115	.002723
1111110100	.001582	.001006	.000889	.001743
1111110011	.002482	.002438	.002115	.002749
1111110010	.001572	.001012	.000889	.001762
1111110001	.001547	.001016	.000889	.001751
1111110000	.001434	.000732	.000539	.002087
1111101111	.006241	.010513	.007744	.008455
1111101110	.002570	.002417	.002115	.002767
1111101101	.002458	.002409	.002115	.002741
1111101100	.001592	.001025	.000889	.001752
1111101011	.002370	.002395	.002115	.002723
1111101010	.001561	.001021	.000889	.001760
1111101001	.001518	.001015	.000889	.001744
1111101000	.001444	.000748	.000539	.002076
1111100111	.002300	.002362	.002115	.002720
REMAINING PATTERNS	.866271	.814880	.882800	.842793
AVERAGE DIFFERENCE	.000218	.000242	.000329	

Dr. Terry Ackerman
American College Testing Programs
P.O. Box 168
Iowa City, IA 52243

Dr. Robert Ahlers
Code N711
Human Factors Laboratory
Naval Training Systems Center
Orlando, FL 32813

Dr. James Algina
University of Florida
Gainesville, FL 32605

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Dr. Eva L. Baker
UCLA Center for the Study
of Evaluation
145 Moore Hall
University of California
Los Angeles, CA 90024

Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Tel Aviv, Ramat Aviv 69978
ISRAEL

Dr. Arthur S. Blaiwes
Code N711
Naval Training Systems Center
Orlando, FL 32813

Dr. Bruce Bloxom
Defense Manpower Data Center
550 Camino El Estero,
Suite 200
Monterey, CA 93943-3231

Dr. R. Darrell Bock
University of Chicago
NORC
6030 South Ellis
Chicago, IL 60637

Cdt. Arnold Bohrer
Sectie Psychologisch Onderzoek
Rekruterings-En Selectiecentrum
Kwartier Koningen Astrid
Bruijnstraat
1120 Brussels, BELGIUM

Dr. Robert Breaux
Code N-095R
Naval Training Systems Center
Orlando, FL 32813

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. Lyle D. Broemeling
ONR Code 1111SP
800 North Quincy Street
Arlington, VA 22217

Mr. James W. Carey
Commandant (G-PTE)
U.S. Coast Guard
2100 Second Street, S.W.
Washington, DC 20593

Dr. James Carlson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd.
Chapel Hill, NC 27514

Dr. Robert Carroll
OP 01B7
Washington, DC 20370

Mr. Raymond E. Christal
AFHRL/MOE
Brooks AFB, TX 78235

Dr. Norman Cliff
Department of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

Director,
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Dr. Stanley Collyer
Office of Naval Technology
Code 222
800 N. Quincy Street
Arlington, VA 22217-5000

Dr. Hans Crombag
University of Leyden
Education Research Center
Boerhaavelaan 2
2334 EN Leyden
The NETHERLANDS

Mr. Timothy Davey
University of Illinois
Educational Psychology
Urbana, IL 61801

Dr. Dattprasad Divgi
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Hei-Ki Dong
Bell Communications Research
6 Corporate Place
PYA-1k226
Piscataway, NJ 08854

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC
(12 Copies)

Dr. Stephen Dunbar
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. James A. Earles
Air Force Human Resources Lab
Brooks AFB, TX 78235

Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. John M. Eddins
University of Illinois
252 Engineering Research
Laboratory
103 South Mathews Street
Urbana, IL 61801

Dr. Susan Embretson
University of Kansas
Psychology Department
426 Fraser
Lawrence, KS 66045

Dr. Benjamin A. Fairbank
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

Dr. Pat Federico
Code 511
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52240

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Fregly
AFOSR/NL
Bolling AFB, DC 20332

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1601 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Dipl. Pad. Michael W. Habon
Universitat Dusseldorf
Erziehungswissenschaftliches
Universitatsstr. 1
D-4000 Dusseldorf 1
WEST GERMANY

Dr. Ronald K. Hambleton
Prof. of Education & Psychology
University of Massachusetts
at Amherst
Hills House
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Dr. Paul W. Holland
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Prof. Lutz F. Hornke
Institut fur Psychologie
RWTH Aachen
Jaegerstrasse 17/19
D-5100 Aachen
WEST GERMANY

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 90010

Mr. Dick Hoshaw
OP-135
Arlington Annex
Room 2834
Washington, DC 20350

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
Department of Education
University of Alberta
Edmonton, Alberta
CANADA

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Robert Jannarone
Department of Psychology
University of South Carolina
Columbia, SC 29208

Dr. Dennis E. Jennings
Department of Statistics
University of Illinois
1409 West Green Street
Urbana, IL 61801

Dr. Douglas H. Jones
Thatcher Jones Associates
P.O. Box 6640
10 Trafalgar Court
Lawrenceville, NJ 08648

Dr. Milton S. Katz
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. G. Gage Kingsbury
Portland Public Schools
Research and Evaluation Department
501 North Dixon Street
P. O. Box 3107
Portland, OR 97209-3107

Dr. William Koch
University of Texas-Austin
Measurement and Evaluation
Center
Austin, TX 78703

Dr. James Kraatz
Computer-based Education
Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Daryll Lang
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Educational Testing Service
Princeton, NJ 08541

Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

Dr. Robert Lockman
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. James Lumsden
Department of Psychology
University of Western Australia
Nedlands W.A. 6009
AUSTRALIA

Dr. Milton Maier
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. William L. Maloy
Chief of Naval Education
and Training
Naval Air Station
Pensacola, FL 32508

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. Clessen Martin
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Dr. James McBride
Psychological Corporation
c/o Harcourt, Brace,
Javanovich Inc.
1250 West 6th Street
San Diego, CA 92101

Dr. Clarence McCormick
HQ, MEPCOM
MEPCT-P
2500 Green Bay Road
North Chicago, IL 60064

Dr. Robert McKinley
Educational Testing Service
20-P
Princeton, NJ 08541

Dr. James McMichael
Technical Director
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Barbara Means
Human Resources
Research Organization
1100 South Washington
Alexandria, VA 22314

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr. William Montague
NPRDC Code 13
San Diego, CA 92152-6800

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Headquarters, Marine Corps
Code MPI-20
Washington, DC 20380

Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Oklahoma City, OK 73069

Deputy Technical Director
NPRDC Code 01A
San Diego, CA 92152-6800

Director, Training Laboratory,
NPRDC (Code 05)
San Diego, CA 92152-6800

Director, Manpower and Personnel
Laboratory,
NPRDC (Code 06)
San Diego, CA 92152-6800

Director, Human Factors
& Organizational Systems Lab,
NPRDC (Code 07)
San Diego, CA 92152-6800

Fleet Support Office,
NPRDC (Code 301)
San Diego, CA 92152-6800

Library, NPRDC
Code P201L
San Diego, CA 92152-6800

Commanding Officer,
Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. Harold F. O'Neil, Jr.
School of Education - WPH 801
Department of Educational
Psychology & Technology
University of Southern California
Los Angeles, CA 90089-0031

Dr. James Olson
WICAT, Inc.
1875 South State Street
Orem, UT 84057

Office of Naval Research,
Code 1142PT
800 N. Quincy Street
Arlington, VA 22217-5000
(6 Copies)

Office of Naval Research,
Code 125
800 N. Quincy Street
Arlington, VA 22217-5000

Assistant for MPT Research,
Development and Studies
OP 01B7
Washington, DC 20370

Dr. Judith Orasanu
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Randolph Park
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Administrative Sciences Department,
Naval Postgraduate School
Monterey, CA 93940

Department of Operations Research,
Naval Postgraduate School
Monterey, CA 93940

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235

Dr. Barry Riegelhaupt
HumRRO
1100 South Washington Street
Alexandria, VA 22314

Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
3108 AustinPeay Bldg.
Knoxville, TN 37916-0900

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152-6800

Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Dan Segall
Navy Personnel R&D Center
San Diego, CA 92152

Dr. W. Steve Sellman
OASD(MRA&L)
2B269 The Pentagon
Washington, DC 20301

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujusawa 251
JAPAN

Dr. William Sims
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. H. Wallace Sinaiko
Manpower Research
and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314

Dr. Richard E. Snow
Department of Psychology
Stanford University
Stanford, CA 94306

Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Paul Speckman
University of Missouri
Department of Statistics
Columbia, MO 65201

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. Peter Stoloff
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

Dr. William Stout
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Maj. Bill Strickland
AF/MPXOA
4E168 Pentagon
Washington, DC 20330

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. John Tangney
AFOSR/NL
Bolling AFB, DC 20332

Dr. Kikumi Tatsuoka
CERL
252 Engineering Research
Laboratory
Urbana, IL 61801

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
University of Missouri
Department of Statistics
222 Math. Sciences Bldg.
Columbia, MO 65211

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E. Street, NW
Washington, DC 20415

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 310
St. Paul, MN 55114

Dr. Frank Vicino
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Howard Wainer
Division of Psychological Studies
Educational Testing Service
Princeton, NJ 08541

Dr. Ming-Mei Wang
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Thomas A. Warm
Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Dr. Brian Waters
Program Manager
Manpower Analysis Program
HumRRO
1100 S. Washington St.
Alexandria, VA 22314

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

Dr. Ronald A. Weitzman
NPS, Code 54Wz
Monterey, CA 92152-6800

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 12
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Rand R. Wilcox
University of Southern
California
Department of Psychology
Los Angeles, CA 90007

German Military Representative
ATTN: Wolfgang Wildegrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational
Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
Psychological Corporation
c/o Harcourt, Brace,
Javanovich Inc.
1250 West 6th Street
San Diego, CA 92101

Dr. Martin F. Wiskoff
Navy Personnel R & D Center
San Diego, CA 92152-6800

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering
Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Kentaro Yamamoto
Computer-based Education
Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
Memory & Cognitive
Processes
National Science Foundation
Washington, DC 20550

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